ON AN ANALYSIS MODEL OF THE SPATIAL ORGANIZATION OF PRODUCTION, BASED ON THE ISOMORPHISM WITH THE PERMUTATION SYMMETRIC GROUP S_n

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Abstract: *The notion of a group, defined as one of the most important generalizing notions of modern mathematics, provides a strong basis to study the problems of organizing production in time and space. The analysis of the group of technological operations based on isomorphism with the symmetric S_n group offers the possibility to assimilate them with this group, ignoring the set and the operation for each group.*

Keywords*:* Group, Subgroup, Permutations, Spatial organization of production, Production link.

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1. INTRODUCTION

The starting point in developing the analysis model of the spatial organization of production is the theory of groups, arising from the need to find a mathematical apparatus necessary to study the symmetry properties of the real world. Indeed, knowing the group properties of some mathematical or physical objects provides important information about the structure of these objects. The fundamental idea in developing the analysis model is that any finite group is isomorphic with a subgroup of a group of permutations, showing that the finite groups sink essentially into the permutations groups.

The possible maximum symmetry of a symmetric group of operations S_3, S_4, ..., S_n generates a nucleus of base operations given by their normal (invariant) subgroup, causing the spatial arrangement of the other operations around the nucleus.

The two cases analyzed in this model, with their subcases, show precisely the transfer of properties from one group to the other isomorphic group; they are considered identical in terms of their algebraic behavior.

2. PROBLEMS REGARDING THE SPATIAL ORGANIZATION OF THE ENTERPRISE PRODUCTION

The spatial organization of production within enterprises includes a set of problems that can be systematized on three main areas, namely:

• locating the production links in the structure of the enterprise;

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• establishing the configuration of technological flows;
• sizing the necessary production areas.

The problems of these three areas are approached both at the general level of the enterprise, and at the structural production links within it. The substantiation solutions for the spatial organization of production are achieved through a specialized study, based on certain systems and methodology. Such a study is used either to design a new production process (when a production unit is put into service or a product is assimilated in the nomenclature), or when the specific conditions require to improve the organization of an existing production process. The sizing methods of production areas presented in this study can also be used in other studies regarding the improvement of ongoing production processes, with some insignificant exceptions (Badea,F. ,2005; Dima,I.C., 2006).

The location of production links aims at the rational arrangement of the different production units of the enterprise on its territory, the workplaces and the workshops within the production departments, according to the characteristics, structure and general organization of the production process.

The location of workplaces and workshops within the production departments determines the existence of three systems of spatial organization of production, which require the following specific principles: the technological principle, the principle per object and the mixed principle.

The system of spatial organization of production based on the location of workplaces according to the technological principle involves, in technological terms, the construction of homogeneous workgroups within the production department or the workshop. Each group of workplaces performs a certain operation on several different types of products from a technological point of view (with ranges and successions of different operations). This location of workplaces is used in individual, small and medium series production, providing the execution of various products. In the case of spatial organization of production in accordance with the requirements of the technological principle, the location study consists in determining the sequence of different groups of workplaces and how they are arranged on the surface of the production department or workshop (Burghelea and Iacob, 2013; Dima, 2007).

The system of spatial organization of production based on the location of workplaces according to the principle per object is characterized by the formation of some production lines within the department or workshop. Each production line comprises several groups of workplaces, that are technologically different, in which various operations are carried out for several types of products, identical or similar, in technological terms. This method of locating workplaces is used in mass production and large series. In the case of spatial organization of production according to the principle per object, the study of workplace location consists in determining their sequence within the production line. Therefore, if one type of product or several types of identical products are made on the production line (having the same range and sequence of operations), the workspace location on the line follows the order required by the sequence of operations. When the production line is running several types of products, technologically similar (the ranges and sequences of operations differ more or less), there are specific methods to establish the order of workplaces within the production line (Grădinaru, 2002; Păunescu and Burghelea, 2010).

The system of spatial organization of production based on the location of workplaces according to the mixed principle means to create both homogeneous working groups and production lines specialized on object**.**

3. ESSENTIAL ELEMENTS ON THE THEORY OF GROUPS. THE SYMMETRIC GROUP OF n DEGREE (S_n)

Considering the basics of the theory of groups, we list strictly the definitions and the theorems underlying this study (Barbilian, 1985; Bușneag, 1994; Kurosh, 1959; Myller Lebedev, 1953; Purdea and Pic, 1982; Șafarevici, 1989).

Definition 1.1 The pair (G, f) is called a group if the following axioms are satisfied:
a) $\forall$ x,y,z $\in$ G, f(f(x,y),z) = f(x,f(y,z))
b) there is e $\in$ G such that $\forall$ x $\in$ G resulting f(x,e) = f(e,x) = x
c) $\forall$ x $\in$ G, there is x' $\in$ G such that f(x,x')=f(x',x) = e
The pair (G,f) is commutative if $\forall$ x,y $\in$ G is satisfied and the commutative axiom, resulting f(x,y) = f(y,x)

Definition 1.2 The cardinal of G (noted |G|) is called the order of the group.
Consider a set M and E (M) the set of all applications f: M → M are called permutations of set M.

Definition 1.3 The bijective applications f: M → M are called permutations of set M.
A group of permutations on set M is a subgroup of group S (M). If M = {1,2, ..., n}, the group S (M) is denoted by S_n and is called the group of *n*-degree permutations.

Definition 1.4 An element of the group of permutations of set M, finite with *n* elements, is called the n-degree permutation.
A *n*-degree permutation is a bijective function σ: {1,2, ... n} → {1,2, ..., n}, being completely determined if for each i $\in$ {1,2, ..., n} its image is known as σ (i) $\in$ {1,2, ..., n.

Definition 1.5 The pair (S_n,∘), called the symmetric group of *n* degree, is a finite group of order n! which checks the axioms of the group as follows:
a) the associativity of the composition on S_n;
b) the existence of the neutral element *e*, as an identical application of set M ={1,2,..n};

Definition 1.6 Let (S_n,.) the symmetric group of *n* degree *n* and the subset A_n= { σ $\in$ S_n| σ even permutation}.

Definition 1.7 Let (G,*) be a group. A non-empty subset H of G, with the property that is stable to the operation * and H with the induced operation is a group, is called a subgroup of group G.
If G is a group and *e* $\in$ G is the neutral element, then the subsets G and {e} of G are called improper subgroups. Any group H of G different from G and {e} is called a proper subgroup.

Definition 1.8 Let (G,·) a group and *x* $\in$ G a fixed element. The subset of G formed by all the integral powers of element *x*, that is <x> ={x^k|k∈Z} is a subgroup of element G, called the cyclic subgroup generated by element *x*.

Definition 1.9 Let (G,·) be a group and *x* $\in$ G fixed. If there is *n* $\in$ N^* cu x'''= *e*, the smallest number n $\in$ N^* with this property, it is called the order of element *x* in group G. If *n* is the order of element *x*, then we note this by ord(x) = n.

Theorem 1.1 Let (G,·) be a group and x $\in$ G an element of *n* degree. Then the cyclic subgroup generated by *x* has the order *n* and is given by the equality <x> = {e,x,x^2,x….,x^{n-1}} and ord(<x>) = n.

Definition 1.10 A group (G,·) is called cyclic if *x* $\in$ G exists so that G = <x> ={x^k|k∈Z} or, more suggestively, if G is generated by one of its elements.

Theorem 1.2 (Lagrange). The order of any subgroup of a finite group is a divisor of the group order.

Definition 1.11 Let G be a group and H its subgroup. H is called the normal (invariant) subgroup if any *x*∈G and *h*∈H resulting xhx^{-1}∈ H.
The subgroups {G} and {e} are invariant subgroups. If the group (G, •) is abelian, any of its subgroups is a normal subgroup. In the symmetric group (S_n, •) of the permutations of n degree

(non-commutative for $n \geq 3$) the alternate subgroup A_n of even permutations is a normal subgroup.

Definition 1.12 Let (G, *) and (G ', o) two groups. A function f: G → G 'with the property f (x * y) = f (x) o f (y), ∀ x, y ∈ G is called a group morphism. A group morphism from a group to itself is called the endomorphism of that group.

Definition 1.13 Let (G, *) and (G ', o) be two groups. A function f: G → G 'which is an invariant group morphism (meaning the inverse function, and the inverse function f-1: G' → G being also a group morphism) is called group isomorphism, called inverse isomorphism. If there is at least one isomorphism between two groups, we say that the groups are isomorphic and we write G ≈ G '.

Theorem 1.3 A group morphism is isomorphism if and only if it is bijective.

Theorem 1.4 (Cayley). Any finite group is isomorphic with a subgroup of a group of permutations. This theorem is very important because it essentially shows that the finite groups sink into the permutations groups, therefore the study of the finite groups is equivalent to the study of permutation subgroups.

4. A MODEL TO ANALYZE THE SEQUENCE OF TECHNOLOGICAL OPERATIONS BASED ON GROUP ISOMORPHISM

The analysis of isomorphic groups reveals that groups exhibit the same properties in terms of algebraic behavior, since the direct isomorphism and the inverse isomorphism transfer all the properties of one group to the other group. All isomorphic groups form a "type" of groups, that may be assimilated to a single one of them, ignoring the set and the operation of each group.

The model to analyze the technological operations of S_n groups is based on the following reasons:

1. Symmetry is the central feature of the analysis model.
2. Any symmetric group of operations S_3,S_4,......S_n has its own symmetry profile, each having a maximum possible symmetry.
3. The normal subgroup generates the nucleus of the base operations, the arrangement of the other operations / workplaces around the nucleus being given by the other subgroups of S_n.
4. In some cases, the isomorphic dihedral group is symmetric with S_3 symmetric group and it is used in the study of the spatial location of workplaces, while in other cases the study of the symmetric group S_4 is made in the opposite direction, starting from the dihedral group, based on the transfer of properties through inverse isomorphism.

Therefore, the analysis model includes the following cases:

Case 1°. The study of the symmetric group of technological operations S_3

Let us consider the symmetric group S_3 of all permutations of three operations. The table of composition operations (product) of permutations is as follows:

o	e	σ	π	α	β	γ
e	e	σ	π	α	β	γ
σ	σ	π	e	γ	α	β
π	π	e	σ	β	γ	α
α	α	β	γ	e	σ	π
β	β	γ	α	π	e	σ
γ	γ	α	β	σ	π	e

From the permutation composition table, the following subgroups of S_3 result: {e},{e, α },{e, β},{e, γ },{e, σ, π }şi {e, σ, π, α, β, γ} which we note with E, H_α, H_β,H_γ, H_σ şi S_3.

The subgroups E and S_3 are improper groups, while the other groups are proper. At the same time, the neutral element of the subgroups coincides with *e*, and for any element in the subgroups, its inverse within the subgroup coincides with its inverse within S_3 group.

By computing the order for each element of S_3, it results:

$\sigma^2 = \pi \neq e$, $\sigma^3 = \sigma^2 \circ \sigma = \pi \circ \sigma = e$

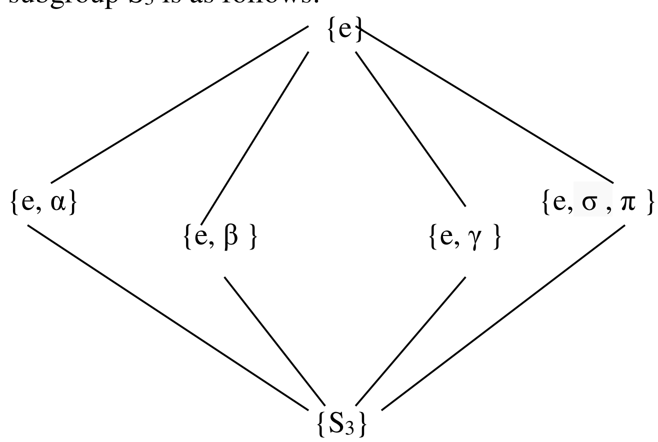
$\pi^2 = \sigma \neq e$, $\pi^3 = \pi^2 \circ \pi = \sigma \circ \pi = e$

$\alpha^2 = \beta^2 = \gamma^2 = e$

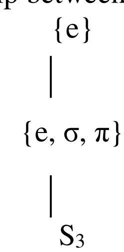
Therefore, ord(α) = ord(β) = ord(γ)=2, ord(σ) = ord(π)=3 and ord(e)= 1 ·

According to Lagrange's theorem, the order of any subgroup of the finite group S_3 is a divisor of the order of that group.

The diagram of subgroup S_3 is as follows:



The description of the inclusion relationship between the normal subgroups of S_3 is as follows:



The normal subgroups {e} and{e, σ, π} generates the nucleus of base operations.

The spatial location of workplaces can also be emphasized by the isomorphism between the group of rotations of an equilateral triangle and the subgroup {e, σ, π}, given that the finite groups sink into the permutation groups (Cayley). By marking a_0, a_1, a_2 the rotations of the equilateral triangle around its center with 0^0,120^0, şi 240^0, the table of the sum operation of two turns (successive execution) is as follows:

	a_0	a_1	a_2
a_0	a_0	a_1	a_2
a_1	a_1	a_2	a_0
a_2	a_2	a_0	a_1

The isomorphism of the two groups is easily observed: although they are constructed from different elements, they have the same structure, their tables differ only as notations and, with the change of the notations, they can be made to coincide perfectly.

Case 2^0. The study of the symmetric group of technological operations S_4

In this case, we study the symmetric group S_4 using the isomorphic groups regarding the symmetric transformations of some geometric figures, as we have shown in the reasoning underlying the analysis model.

Subcase 2.1^0. Study based on the isomorphism with the group of rotations of a cube

Among the different rotations of identical transformation invariant to the K-cube we distinguish:

- 2p / 3 and 4π / 3 corner rotations around a cube diagonal, eight such transformations altogether;
- the rotation of the angle π around a straight line passing through the opposing edge, in total six such transformations;
- the rotations of angles π / 2, π and 3π / 4 around a straight line passing through opposite face centers, in total nine such transformations.

If the identical rotation is added, 24 transformations are admitted.

Now, let us highlight the number of subgroups:

- 6 cyclic subgroups of order 2;
- 4 cyclic subgroups of order 3;
- 3 cyclic subgroups of order 4.

The following subgroups show a particular interest:

- the subgroup of order 12, which consists of 2×4 non-identical rotations around the diagrams, from three rotations with the angle π around the axes connecting the opposite faces, and the identical rotation;
- 3 subgroups of order 8, which consist of those rotations of the cube which transform into itself one of the straight lines joining the centers of two opposing sides. None of these subgroups is a normal subgroup;
- 1 subgroup of order 4, consisting of the identical transformation and three rotations with the π angle around each of the axes joining the centers of the two opposing faces, and which is the only normal group.

This normal subgroup generates the nucleus of the base operations, while the other workplaces / operations may locate around the nucleus in two variants:

- by arranging the cyclic subgroups of orders 2, 3 and 4, according to the specific production conditions;
- by placing the subgroups of orders 8 and 4, considering the first variant of the specific production conditions.

Subcase 2.2^0 The study based on the isomorphism between the tetrahedron rotation group and the alternate group of four-element permutations

Each admitted transformation of tetrahedron means a permutation of its peaks, namely the indices 0, 1, 2, 3; there are 24 permutations of 4 elements in total. Only 12 of them are completed when moving the tetrahedron in space.

Each side median corresponds to two admitted non-identical transformations, namely the rotations around it, with the angles 2π / 3 and 4π / 3. These 11 rotations together with the identical transformation give the 12 transformations admitted by the tetrahedron.

The following subgroups of the tetrahedron rotation group can be identified:

a) improper subgroups

- the whole group considered;
- the subgroup composed only of the neutral element.

b) proper subgroups

- the subgroup H_o of order 4, consisting of the rotations with the angle π around the three edge medians and the identical rotation;
- 3 subgroups of order 2 of H_o, consisting of the rotations with the angles 0 and π around each edge median, noted H_{o1}, H_{o2} and H_{o3};
- 4 subgroups of order 3, each of them consisting in three rotations with the angles 0, 2π/3 and 4 π /3 around the corresponding face madian, noted Hi (i = 0,1, 2, 3).

One may notice that the group of tetrahedron rotations is non-commutative. The only normal group in the tetrahedron rotation group is H_o, and the explanation is: since the group of tetrahedron rotations is isomorphic with the alternate group A_4 of even permutations of four elements, it results that the alternate group of permutations of four elements admits a normal subgroup of order 4. This is very important because for n> 4, the alternate group A_n of *n* element permutations does not contain any normal subgroups, except for the two subgroups that are inappropriate.

As a consequence of the isomorphism with the tetrahedron rotation group, which transfers the properties of this group to the symmetric group of operations S_4, the normal subgroup H_o generates the nucleus of the base operations, while the other operations / workspaces locate around the nucleus through the subgroups of orders 2 and 3 presented above, in close connection with the specific conditions of the company's production.

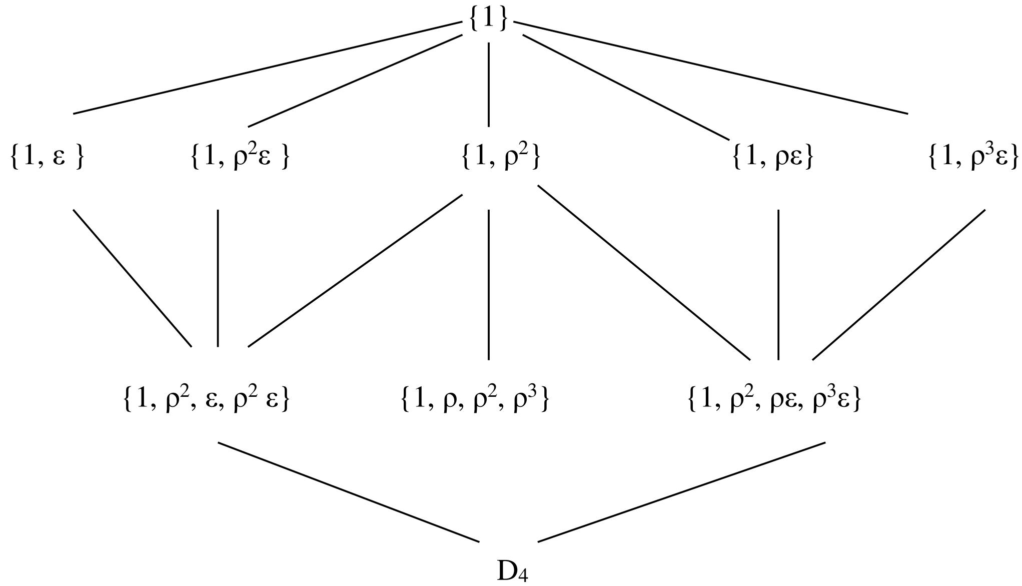
Subcase 2.3. Study based on isomorphism with the dihedral group D_4.

It is known that the dihedral group Dn can sink into the symmetric group Sn and it has at most 2*n* elements, namely | Dn | = 2n. Taking into account the rotation of the dihedral ρ around the center O and the symmetry ε relative to one of the symmetry axes, the group multiplication table $D_4 = \{1, \rho, \rho^2, \rho^3, \varepsilon, \rho\varepsilon, \rho^2\varepsilon, \rho^3\varepsilon\}$ is as follows:

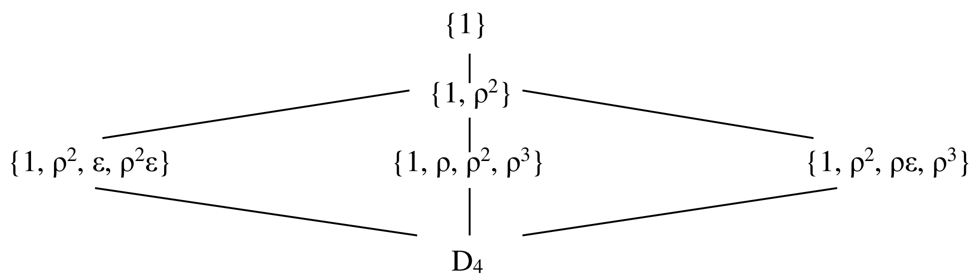
·	1	ρ	ρ^2	ρ^3	ε	$\rho\varepsilon$	$\rho^2\varepsilon$	$\rho^3\varepsilon$
1	1	ρ	ρ^2	ρ^3	ε	$\rho\varepsilon$	$\rho^2\varepsilon$	$\rho^3\varepsilon$
ρ	ρ	ρ^2	ρ^3	1	$\rho\varepsilon$	$\rho^2\varepsilon$	$\rho^3\varepsilon$	ε
ρ^2	ρ^2	ρ^3	1	ρ	$\rho^2\varepsilon$	$\rho^3\varepsilon$	ε	$\rho\varepsilon$
ρ^3	ρ^3	1	ρ	ρ^2	$\rho^3\varepsilon$	ε	$\rho\varepsilon$	$\rho^2\varepsilon$
ε	ε	$\rho^3\varepsilon$	$\rho^2\varepsilon$	$\rho\varepsilon$	1	ρ^3	ρ^2	ρ
$\rho\varepsilon$	$\rho\varepsilon$	ε	$\rho^3\varepsilon$	$\rho^2\varepsilon$	ρ	1	ρ^3	ρ^2
$\rho^2\varepsilon$	$\rho^2\varepsilon$	$\rho\varepsilon$	ε	$\rho^3\varepsilon$	ρ^2	ρ	1	ρ^3
$\rho^3\varepsilon$	$\rho^3\varepsilon$	$\rho^2\varepsilon$	$\rho\varepsilon$	ε	ρ^3	ρ^2	ρ	1

The orders of the elements of D_4 are: ord (1) = 1, ord (ρ) = 4, ord (ρ2) = 2, ord (ρ3) = 4, ord (ε) ord (ρ3ε) = 2.
The subgroups of order 2 of D_4 are {1, ρ2}, {1, ε}, {1, ρε}, {1, ρ2ε}, {1, ρ3ε}.
The subgroups of order 4 of D_4 may be cyclic, the only cyclic subgroup being {1, p, p2, p3}.

The chart of subgroups of D_4 group is the following:



The description of the inclusion relations between the normal subgroups of D_4 is the following:



The three subgroups of order 4 are normal, as the subgroup of order 2, {1, ρ2}, this being the intersection of any two of the subgroups of order 4.

In this subcase, the normal subgroups generate four nuclei of base operations, resulting in four variants of spatial organization of other operations / workplaces around the nuclei.

Adopting one of the four variants should start from the following requirements:

- establishing technological flows without crosses and turns;
- using efficiently the existing production areas;
- moving the resulting materials and products on distances as small as possible;
- minimizing the frequency of transports;
- ensuring a flexible location.

Compliance with these requirements determines the improvement of the indicators that characterize the production activity of the enterprise as follows: reducing the production cycle, increasing labor productivity, reducing production costs, accelerating the rotation speed of the circulating assets and increasing the utilization rate of the existing production areas.

5. CONCLUSIONS

The direct isomorphism and the inverse isomorphism used in this analysis model for the groups S_3 and S_4 of technological operations with the isomorphic groups related to symmetric transformations of some geometric figures show the existence of several nuclei of base operations, depending on the number of invariant subgroups. Symmetry is an important characteristic of this analysis model, emphasizing the importance of the normal (invariant) subgroup and its descendants; therefore, an entire genealogy of maximal normal subgroups may be recorded.

Changing the abstract point of view with the spatial representation of the technological spaces, one may notice that the location of the workplaces in a stable form within the production organization systems cannot meet all the requirements related to the permutation of the operations. Even more, the range of operations of similar products, in technological terms, is changing. As the number and range of operations diversify, the isomorphism provides only an overview of the complexity of the spatial and temporal representation of production.

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