THE APPLICABILITY OF THE UNIFACTORIAL MODEL FOR BRD SHARES QUOTED ON THE BUCHAREST STOCK EXCHANGE

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Abstract: *The single index model or one factor model was generated by William Sharpe (1963), who developed his research based on the idea of simplifying the Markowitz portfolio selection model. Sharpe has proposed a solution whose essential feature is to suppose that the returns of different financial titles are linked exclusively to one another through their common relationship with a basic factor. This purely empirical hypothesis has subsequently become of considerable importance. The Sharpe hypothesis can be formalized by the simple linear regression model. Thus, the profitability of each securities is considered to have only one exogenous determinant, common to all values, usually the general index of the stock market. All other factors that can cause changes in return on a value are specific (endogenous) factors for the investment projects managed by the company issuing the security. In this article we tested the applicability of the market model for the shares issued by BRD - Groupe Societe Generale S.A.*

Keywords*:* Single index model, Systematic risk, BET index.

JEL Classification Codes: G12.

1. INTRODUCTION

The objective of any investor in a financial market is to obtain a certain return on capital employed. However, taking into account any financial gain implies the acceptance of a certain degree of risk or uncertainty in relation to the actual realization of this gain. The relationship between the expected gain of an investment and its level of risk is viewed as directly proportional.

The return actually obtained can be more or less different from the expected one. Thus, the risk of an investment, such as that of a financial asset, can be quantified through the variation of the returns on a certain period around the average value. This variability of the return on an asset within a specified period is given by the standard deviation of the series of rates of return of the respective asset on a certain number of sub-periods.

The total risk of the security has two components: the unsystematic risk, due to specific events of the issuing company, and a capital market risk as a whole. The market risk, also known as systemic risk, is related to the stock market movement and is consequently, unavoidable, being imposed to all investors, while the specific or diversifiable risk can be eliminated by diversification (Markowitz, 1959).

The chart shows the reduction of the unsystematic risk by adding securities in the portfolio. Empiric studies showed that the specific risk can practically be eliminated from portfolios comprising 20-30 randomly selected securities (Stancu, 1997). The systematic (undiversifiable) risk can be assess by applying a single index model or the market model, due to William F.

Sharpe's research in 1963. The undiversifiable risk corresponds to the beta value which, in theory, is a measure of anticipating the volatility of a share, in relation to the overall market.

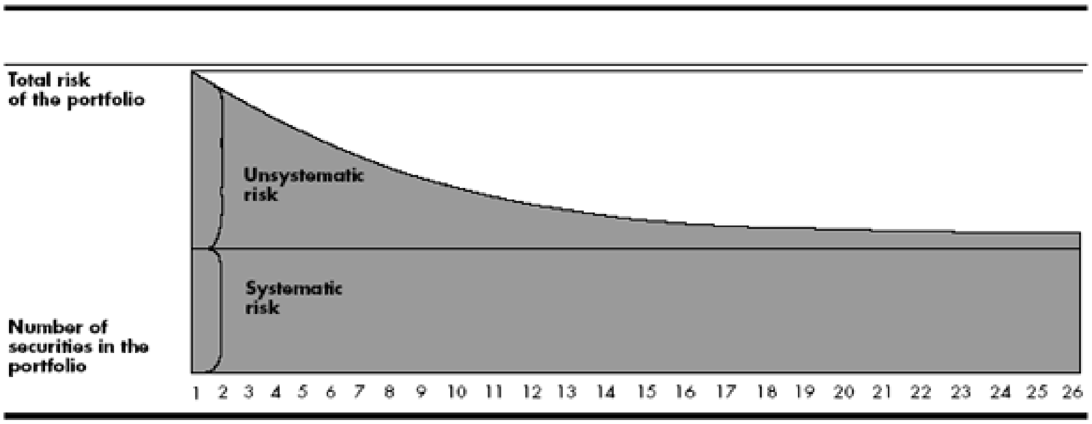


Figure 1: Reduction of unsystematic risk through diversification

Source: Stancu, 1997, pp. 187

The beta coefficient is given by the ratio between the covariance of the security in with the market M and the market variation. The variation of the market return represents the uncertainty related to the impact of economic events in the overall market.

The model proposes a new method for the valuation of the financial assets based on objective financial market criteria, highlighting the calculation formula of the conditional return of a share, which consists of a simple linear regression between the return on a share and the general return of the market, the explanatory factor being the market return.

Consequently, the single index model sets a linear relationship between the return on a share and the market return. This model is not based on any theoretic construction, being a strictly empirical formulation proposed for the first time by Sharpe (1963). Its relevance is due to the fact that it allows for the assessment of the beta of the securities.

$$R_{i,t} = \alpha_i + \beta_i R_{M,t} + \varepsilon_{i,t}$$

where:

$R_{i,t}$ = the return on the security i at time t;

$R_{M,t}$ = the general market return at time t;

β_i = the volatility coefficient, expressing the sensitivity of the share i in relation to the general market return;

α_i = the characteristic coefficient of the analysed security, quantifying the influence of all stable determining factors of the return, except the general market return

$\varepsilon_{i,t}$ = the residual term at time t, quantifying the influence of the factors with random action, characterized by null average and constant dispersion

The purpose of this article is to test the validity of the market model for the shares of BRD- Groupe Societe Generale S.A. listed on the Bucharest Stock Exchange. The assessment of the linear regression model is made based on the data for the two characteristics, recorded in the past.

2. METHODOLOGY

The study uses the weekly returns of the BRD share and of the BET index in the 3 January 2013 – 6 December 2017 period. They were calculated based on the closing prices. Returns calculated using a longer time period (e.g.monthly) might result in changes of beta over the examined period, thereby, introducing biases in beta estimates. On the other hand, high frequency data such as daily observations covering a relatively short and stable time span can result in the use of very dissipated data and thus yield inefficient estimates.

Each series consists of 200 observations of the weekly closing prices. The size of the data set is based on the assumption that we have sufficient information to effectively assess the market model, and to have, in the same period of time a reasonable duration. Thus, in the selection of the time horizon, it was necessary to make a compromise between the sufficient observations for the elimination of the impact of the random rates of return and the excessive duration in which the structure would have changed its configuration.

As a benchmark for the market return, we used the BET index. This is a weighted index that reflects the general trends of the Romanian stock market.

The data were obtained from ten database of BVB and KDT Invest and the returns on the shares are adjusted for dividends, assuming that the dividends are immediately reinvested. The return on the BRD share is calculated as the difference between se the logarithm of two consecutive closing prices of the share

$$R_{it} = \ln(Pit/Pit-1) = \ln(Pit) - \ln(Pit-1)$$

and the return of the BET index as the difference between the logarithm of two consecutive closing values of the index

$$R_{Mt} = \ln(PMt/PMt-1) = \ln(PMt) - \ln(PMt-1)$$

The parameters of the linear model are usually determined by using the method of least squares. The minimum of the difference squares is obtained in the point where the derivative of the function σ_i^2 in relation to R_M is equal to 0.

$$\sum (R_{i_t} - \overline{R_i})^2 = \sum \left[R_{i_t} - \left(\alpha_i + \beta_i * R_{M_t}\right)\right]^2 \text{ - MIN}$$

From the calculation of the derivative and by equating it with 0, $\Rightarrow$

$$\beta = \frac{\sum \left(R_{it} - \overline{R_i}\right)\left(R_{Mt} - \overline{R_M}\right)}{\sum \left(R_{Mt} - \overline{R_M}\right)^2} = \frac{\sigma_{iM}}{\sigma^2_M}$$

The equation for the intersection of the regression curve, α is:

$$\alpha = \overline{R}_M - \beta \overline{R}_i$$

By using the method of least squares, the values of the resulting characteristic are assessed based on the formula:

$$\hat{R}_t = \alpha + \beta R_{M,t}$$

where α and β are the estimators of the parameters of the regression line.

The real values of the resulting characteristic are equal to the assessment obtained by means of the regression model, corrected with the residual error:

$$R_t = \hat{R}_t + \varepsilon_t$$

In defining the linear regression function, the most frequent assumptions taken into account are as follows (Gujarati, 1978):

- the data series that are not affected by registration errors;
- for each fixed value of the factorial characteristic, the residual variable is zero average;
- the lack of correlation of residues expresses the fact that between the residual terms there is no covariance phenomenon;
- homoscedasticity (constant variance) of the residual factors;
- the hypothesis of the non-correlation of the residual variable with the independent one, which assumes that cov(R_{M},, ε_i) = 0 the factorial (explicative) variable is not correlated with the residual variable;
- the model's errors are normally distributed according to a co-distribution of a zero mean and a dispersion $\sigma 2$

3. ANALYSIS OF THE RESULTS

As a first stage of the analysis of the series of data taken into consideration, we used the Eviews software package in order to generate a series of statistics typical for each of the two indicators taken into account: the BET index return and the return on the BRD shares.

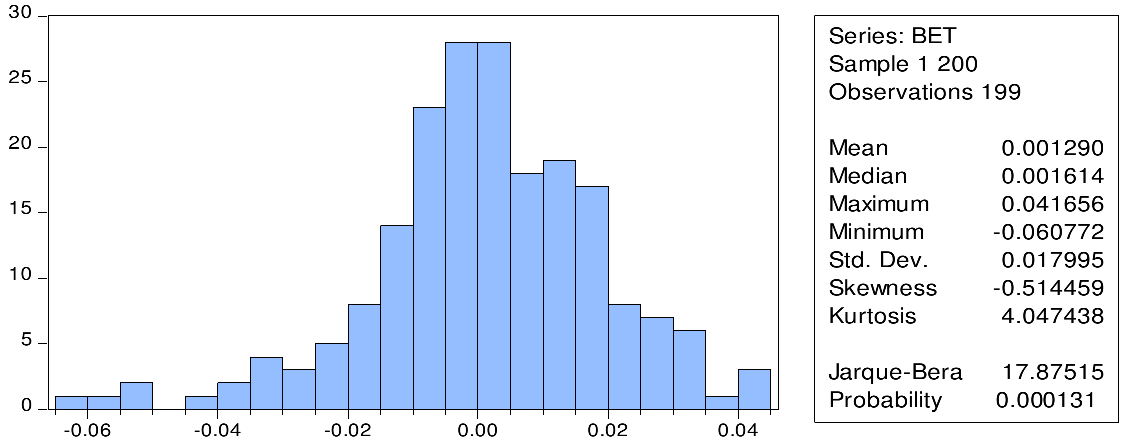


Figure 2: The histogram and the descriptive statistics of the BET index return
Source: Authors' calculations based on Eviews data

The statistical tests applied to the series of data related to the weekly return of the BET index showed that its average value is 0,129%, and the distribution of thus data series does not correspond exactly to the normal distribution.

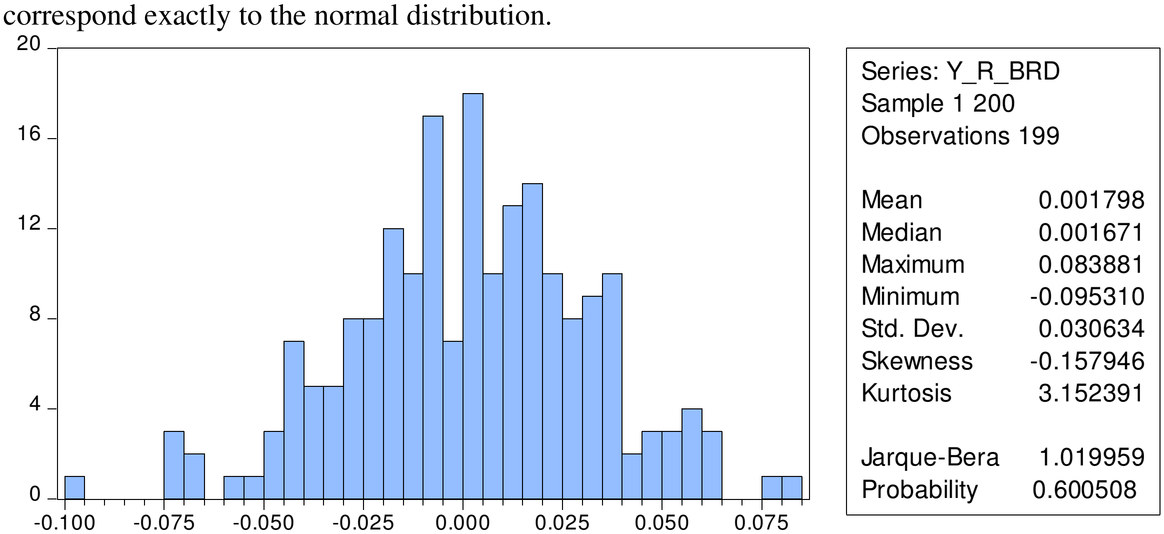


Figure 3: The histogram and the descriptive statistics of the BRD shares
Source: Authors' calculations based on Eviews data

In the case of the analysis of the series of data related to the evolution of the weekly return on the BRD shares, it is important to specify that this indicator records an average value of 0.1798%, and the recorded values follow the normal distribution.

The absence of measurement errors in the noticed values was proven by verifying the fitting of the real values in the following ranges: $R_{BET} \in (\overline{R}_{BET} - 3\sigma; \overline{R}_{BET} + 3\sigma)$ and $R_{BRD} \in (\overline{R}_{BRD} - 3\sigma; \overline{R}_{BRD} + 3\sigma)$, calculated based on the descriptive statistics of R_{BET} și R_{BRD}. In order to highlight the type of link between the two data series, I made the graphic representation of the pairs of points that include the weekly returns on the BET index and on the corresponding BRD share.

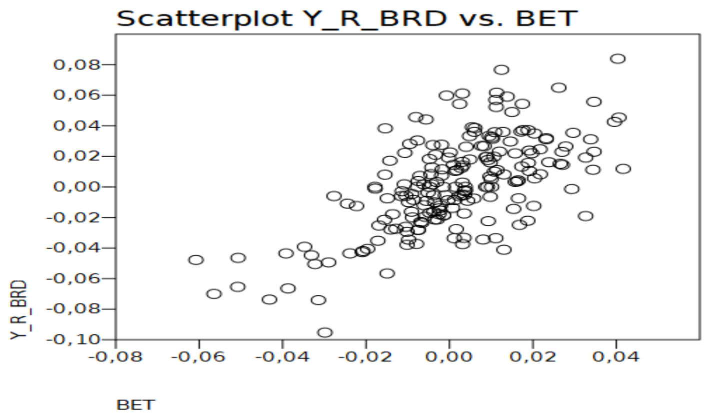


Figure 4: The correlation diagram of the returns on BET/BRD
Source: Authors' calculations based on Eviews data

The chart supports the existence of a direct and linear link between the return on the BRD shares and on the BET index, i.e. $R_{BRD} = \alpha + \beta R_{BET} + \varepsilon_t$.

R_{BRD}- is the dependant variable (explained, endogenous, resultative),
α - intercept (the constant term),
β - the slope of the regression line,
R_{BET} – the vector of the independent variable (explicative, exogenous),
ε - a variable, interpreted as error (measurement error).

Based on the above-mentioned elements, by means of a software package, the parameters of the regression model presented hereinabove were estimated using the method of least squares.

Table 1. Characteristics of the regression model

Dependent Variable: Y_R_BRD
Method: Least Squares
Date: 08/26/18 Time: 10:06
Sample (adjusted): 1 199
Included observations: 199 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000373	0.001661	0.224333	0.8227
BET	1.104884	0.092275	11.97379	0.0000

R-squared	0.421221	Mean dependent var	0.001798
Adjusted R-squared	0.418283	S.D. dependent var	0.030634
S.E. of regression	0.023365	Akaike info criterion	-4.665192
Sum squared resid	0.107543	Schwarz criterion	-4.632094
Log likelihood	466.1866	Hannan-Quinn criter.	-4.651796
F-statistic	143.3716	Durbin-Watson stat	2.219989
Prob(F-statistic)	0.000000		

Estimation Command:
=========================
LS Y_R_BRD C BET
Estimation Equation:
=========================
Y_R_BRD = C(1) + C(2)*BET
Substituted Coefficients:
=========================
Y_R_BRD = 0.000372515142194 + 1.1048844901*BET

Consequently, the regression function is: $R_{BRD} = 0.000373 + 1.104884\ R_{BET}$

The link between the return on the BRD shares and the return on the BET index is a direct one and can be interpreted as follows: a 1% increase of the index return will lead to an increase of the return on the BRD shares by 1.104%.

The first part of the table contains information related to the coefficients: the second column – the value of the coefficient, Std. Error- the standard error of the coefficient (the standard deviation in the sample distribution of the coefficient), t- the statistics of the significance test of the coefficient and Prob.- the critical probability of the test. A coefficient is significant (different from zero in the regression equation) if Prob< the chosen level of significance (in this case 5%).

The correlation coefficient (r) is obtained by extracting the square root of R Square, and it is 0.649, fitting in the 0.6-0.8 range corresponding to a high correlation. The R-squared and the adjusted R-squared shows the extent to which the return on the BRD shares is explained by the independent variable. These indicators can have values fitting in the range [0.1]. The closer the values to 1, the better the model. Approximately 42% of the variation of the return on the BRD shares is explained through the chosen linear regression model.

Another important piece of information is the F statistic by means of which the significance of the independent variable is tested. The probability associated to the test is 0.000 which leads us to rejecting the hypothesis of the lack of significance of the independent variable in favour of the hypothesis that the regression model is a significant one. (the chosen model adjusts the data in the sample adequately). Standard Error is the standard error that shows the average size of the deviation of the noticed values y_t from the theoretical values found on the regression line, $\hat{y}t$ (in such case ±0.023365).

We will test below the significance of the parameters through the student test.

We have the hypotheses:

- null hypothesis, *H0* : $\alpha = 0$ or $\beta = 0$,
- Alternative hypothesis, *H1*: $\alpha \neq 0$ or $\beta \neq 0$,

The coefficient of the return on the BET index is $\beta = 1.104884$, the standard error *Std. Error* (β) *=0.092275, t statistic expressed by* $t_\beta = 11.97$, *the probability value 0.000* which shows us that the return on the index is an important influence factor of the return on the BRD shares.

The coefficient of the free term in the regression model is $\alpha = 0.0000373$ the standard error *Std. Error* (α) = 0.001661, t statistic expressed through $t\alpha = 0{,}22$ having the probability value 0.8227, therefore free term is not significant for the chosen regression model.

The predicted Y value ($\hat{y}t$) for the return on the BRD shares is obtained replacing the values X of the return on the BET index in

$YX = 0.0000373 + 1.104884\ X$, the assessed model.

From the calculations made, we found that the sum of the adjusted values is equal to the sum of the empirical values, which enables us to say that the assessment of the parameters of the regression equation is correct.

The value of the prediction error (Residuals) is obtained as the difference between the observed value (actually recorded by the return on the BRD shares) and the forecast value.

In order to observe all the basic hypotheses in the application of the linear regression, we will apply the following tests:

- Jarque-Bera in order to prove that the distribution of errors is normal;
- Durbin – Watson in order to check the lack of correlation between the residual factors;
- The White test in order to verify the homoscedasticity of errors.

The Jarque-Bera test presents the following hypotheses:

H0: the errors follow a normal distribution: skewness = 0 and kurtosis = 3

H1: the errors don't follow a normal distribution

The Jarque-Berra is an asymptotic test, which follows a chi squared distribution with a number of degrees of freedom equal to 2, having the following form:

$$\text{Jarque-Bera} = \frac{N}{6}\left(S^2 + \frac{(K-3)^2}{4}\right)$$

where:

N = the number of observations;

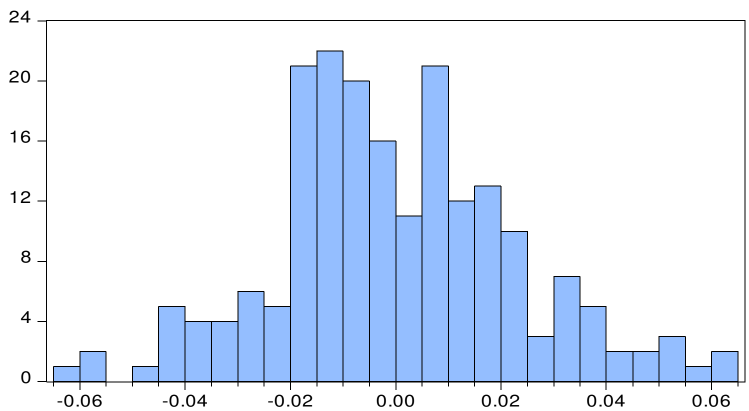
S = the asymmetry coefficient (skewness), measuring the symmetry of the distribution of errors around their average

K = the flattening coefficient calculated by Pearson (kurtosis), which measure the distribution arch (how "sharp" or flattened the distribution is compared to the normal distribution)

The Jarque-Berra test is based on the hypothesis that the normal distribution has a skewness coefficient equal to zero, S = 0, and a flattening coefficient equal to three, K = 3.

If the probability p(JB) corresponding to the calculated value of the test is sufficiently low, then the hypothesis of the normality of errors is rejected, while, on the contrary, for a sufficiently high level of the probability, the hypothesis of the normality of errors is accepted, or if JB> $\chi2$,then the hypothesis of normality of errors is rejected.

The calculation made by the Eview software for the statistics of the Jarque-Berra test confirms the value 1.931276, a value lower than the critical value of the Jarque-Berra[1] test, and the probability associated to the JB test is higher than the level of significance selected (0.38074>0.05). For this reason, we accept the null hypothesis, namely that this regression follows a normal distribution. This can also be seen in images:



Series: Residuals	
Sample 1 199	
Observations 199	
Mean	-1.88e-18
Median	-0.001980
Maximum	0.062488
Minimum	-0.062702
Std. Dev.	0.023305
Skewness	0.226068
Kurtosis	3.168791
Jarque-Bera	1.931276
Probability	0.380740

Figure 5: Histogram and descriptive statistics of the residues

Source: Authors' calculations based on Eviews data

From the histogram and the descriptive statistics of the residual variable we can see that it records an average value of approximately 0 ($-1.88 * 10^{-18}$) and follows the law of normal distribution with a skewness coefficient approximately equal to zero (skewness = 0.226068) and a flattening coefficient approximately equal to three (kurtosis = 3.1687)

1 According to the distribution χ^2, the critical value of the Jarque-Berra test is 5.99 for a statistical significance threshold of 0.05

The Durbin –Watson test will be applied to highlight the lack of correlation of the residues. This test is an important step in the validation of the model because the autocorrelation of the errors affects the quality of the estimators.

The Durbin – Watson statistical test uses the pair of hypotheses:
*H*0: ρ = 0 (null hypothesis);
*H*1: ρ ≠ 0 (alternative hypothesis).

In order to test the null hypothesis, the empirical term is calculated by means of the following formula:

$$DW = \frac{\sum_{i=2}^{n} (\hat{\varepsilon}_{it} - \hat{\varepsilon}_{it-1})^2}{\sum_{i=1}^{n} \hat{\varepsilon}_{it}^2}$$

The DW statistics is tabulated, and its values depend on the specified significance level, the number of observations in the sample and the number of influence variables in the regression model. The latter has, for a specified level of significance, two critical values obtained from the tables: DL and DU.

The range that allows for accepting the null hypothesis is: if $DW \in$ (DU,4 -DU) there is no autocorrelation or lack of first-order autocorrelation of the errors.

In the analysed model, the DW statistic is 2.219989. For a level of significance of 5%, a number of 200 observations and an influence variable, the tabulated values of the statistics are: DL = 1,758 and DU = 1,779. The value obtained in the model belongs to the range (1.779; 4-1.779) so the null hypothesis according to which there is no phenomenon of autocorrelation of the errors is accepted.

The verification of the hypothesis of homoscedasticity of the errors in the case of this model will be made by means of the White test. The homoscedasticity refers to the hypothesis of the regression model that argues that the errors of the model must have the same variance.

The application of the White test implies following the stages below:

- The assessment of the parameters of the initial model and the calculation of the assessed values of the residual variable,
- Building an auxiliary regression, based on the assumption of the existence of a dependence relationship between the square of the error values, the exogenous variable included in the initial model and the square of its values:

$$\varepsilon^2 = a_0 + a_1 R_{BET} + a_2 R^2_{BET} + u$$

From the auxiliary regression we retain the determination coefficient. The White statistic: $W = nR^2$ follows asymptotically a distribution χ^2 with the degrees of freedom given by the number of regressors in the auxiliary equation date.

If the calculated value of the W statistic, i.e. W calculated = $nR^2 > \chi^2$ critical, or if the probability is lower than the level of significance selected, we reject the null hypothesis (the homoscedasticity of the errors) and we calculate the alternative hypothesis (the random errors are heteroscedastic).

The White test was generated by means of the Eviews software package:

Table 2. Characteristics of the auxiliary regression model

Heteroskedasticity Test: White
Null hypothesis: Homoskedasticity

F-statistic	0.306106	Prob. F(2,196)	0.7367
Obs*R-squared	0.619647	Prob. Chi-Square(2)	0.7336
Scaled explained SS	0.658504	Prob. Chi-Square(2)	0.7195

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 08/26/18 Time: 10:13
Sample: 1 199
Included observations: 199

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000541	6.69E-05	8.092117	0.0000
BET^2	-0.011461	0.105609	-0.108519	0.9137
BET	0.002374	0.003239	0.732884	0.4645

R-squared	0.003114	Mean dependent var	0.000540
Adjusted R-squared	-0.007059	S.D. dependent var	0.000798
S.E. of regression	0.000801	Akaike info criterion	-11.40726
Sum squared resid	0.000126	Schwarz criterion	-11.35761
Log likelihood	1138.022	Hannan-Quinn criter.	-11.38717
F-statistic	0.306106	Durbin-Watson stat	1.775153
Prob(F-statistic)	0.736660		

From the analysis of the results we retained the following information Obs*R-squared = 0.619647 and Prob 0.7336. For a significance threshold of 0.05 the tabulated value of the test χ^2 with 2 degrees of freedom is 5.99, the White test statistic being 0.619647, it highlights that the random errors are homoscedastic (Obs*R-squared < χ^2). Moreover, the acceptance of the null hypothesis (the homoscedasticity of the errors) is supported by the fact that the probability associated to the test is higher than the selected significance threshold ($\alpha = 0.05$).

4. CONCLUSIONS

The market model revolutionised the portfolio management methods, proposing the first quantified approach of risk. Its use grew significantly, because most investors study profitability and the risk of a portfolio in relation to the behaviour in the market in general. However, the model is a simplification of reality, due to the fact that it tries to explain the variations of the prices of securities based on a single identical factor for all securities. Moreover, the beta of an action, measured in the past year, doesn't always provide sufficient clues in relation to the sensitivity of a security in relation to the market movement in the following year, the assessments, generally made in a previous period, are simplified, a fact proved by experience.

The variability of the regression model is confirmed by the values of the tests performed by us:

- the Fisher test for the validation of the significance of the model. The probability associated to the test is 0.000 which leads us to reject the hypothesis of the lack of significance of the independent variable in favour of the hypothesis that the regression model is significant.

- the t- Student test applied with the purpose of testing the influence of the parameters. Thus, for the coefficient of the return on the BET index BET ($\beta = 1.1048840$), the probability associated to the t statistic, expressed through $t_\beta = 11.97$, is 0.000 which shows that the return on the index is an important influence factor of the return on the BRD shares. For the coefficient of the free term ($\alpha = 0.0000373$), the t statistic expressed through $t\alpha = 0.22$ has a value of the probability of 0.8227, showing that the free term is not significant for the regression model.

- the Jarque-Bera test in order to prove that the distribution of the errors is normal. The calculation made by the Eview software for the statistic of the Jarque-Berra test is 1.931276, a value lower than the critical value of the Jarque-Berra test of 5.99 for a statistic significance threshold of 0.05, leading to the acceptance of the hypothesis of the normality of the errors.

- the Durbin – Watson test in order to verify the lack of correlation between the residual factors. The DW statistic is 2.219989 for the analysed model, a value that fits in the range = (DU; 4 -DU) and (1.779; 4-1.779) respectively, which allows for the acceptance of the null hypothesis: the lack of autocorrelation of the errors.

- the White test with the purpose of verifying the homoscedasticity of the errors. The calculated value of the W statistic is $0.619647 < 5.99$ the tabulated value of the test χ^2 with 2 degrees of freedom is 5.99 for a significance threshold of 0.05 which led us to accept the null hypothesis, the homoscedasticity of the errors.

The profitability of the stock market, represented by the BET index, is an important factor for the evolution of the return on the BRD shares. A 1% increase in the weekly return on the index will lead to a 1.104% increase in the weekly return on the BRD shares. The value of the determination coefficient R -squared of 0.4212 suggests that approximately 42% of the variation of the return on the BRD shares is explained by the unifactorial model.

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