THE USE OF MATHEMATICAL MODELS FOR THE OPTIMIZATION OF TRANSPORTS FROM THE COMPANY'S LOGISTICS AND OF FORRESTER MODELING TECHNIQUES ON TRAFFIC PROBLEMS

Doruleţ GRADINARU[1], Puiu GRADINARU[2]

University of Pitești, gradinaru_dorulet@yahoo.com
University of Pitești, gradinarupuiu@yahoo.com

Abstract: *The optimization of transport from the company's external logistic entails, besides the use of mathematical models based on linear programming, a correlation with the system's dynamic functioning models. The purpose is minimizing delays in road traffic, thus having the material resources present in time for the production process. The use of Forrester techniques for this optimizes the ratio between the demands of the market for different products made by the company and the means of supplying them.*

Key words: dynamic models; Forrester approaches; exponential delays; reverse connection; dynamic system behavior.

JEL Classification Codes: C6, M1.

1. INTRODUCTION

The optimization of domestic and international road transport services is an extremely important element if transport costs are to be reduced.

A well-established logistics of transportation can save a large part of the expenses and can evolve to the development of the business. A transport company can provide tailor-made solutions and optimize the logistics process in collaboration with customers so that the transport costs are as low as possible.

The most commonly used transport optimization mathematical models of logistics in transmission and distribution networks use the simplex method, which starts from the determination of an initial basic solution. This is the reason why this mathematical model is presented at the beginning. The research methodology used starts from Forrester's dynamic models, more precisely from the concept of exponential delay, which is the starting point in creating a model for car traffic modeling. The variables of the proposed model are continuous and differential space-time functions, and traffic variations are assumed to arise as a result of the random disturbance of uniform traffic.

2. THE MATHEMATICAL MODEL FOR OPTIMIZING TRANSPORTS FROM THE COMPANY'S LOGISTIC

The mathematical model for transport problems entails "m" supply centers $A_1, A_2, ..., A_m$ in which a product is available in $a_1, a_2, ..., a_m$, quantities, "n" consumption centers $B_1, B_2, ..., B_n$ where that product is necessary in quantities $b_1, b_2, ..., b_n$, and the unitary costs for transportation

c_{ij}. The task is to determine the product quantities that will be transported from each supplier to each beneficiary in such a way for the total of the transportation costs to be minimal.

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We presume that the following conditions are met:

- the total quantity of product moved from supplier A_i to the "n" beneficiaries B_1, B_2, ..., B_n is at most equal to that available in A_i:

$$x_{i1} + x_{i2} + \cdots + x_{ir}, \qquad i = 1, 2, \quad (1.1)$$

- the total quantity received by beneficiary B_j from those "m" suppliers A_1, A_2, ..., A_m is at most equal to the need of B_j:

$$x_{1j} + x_{2j} + \cdots + x_{m}, \qquad j = 1, 2 \quad (1.2)$$

- the transported quantities need to be positive.

$$x_{ij} \geq 0, \quad i = 1, 2, \dots, m, \quad j = 1, 2 \quad (1.3)$$

The mathematical model of the transportation problem will be:

$$[min] f = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

$$\begin{cases} \sum_{j=1}^{n} x_{ij} \leq a_i, & i = 1, 2, \dots, m \\ \sum_{i=1}^{m} x_{ij} \leq b_j, & j = 1, 2, \dots, n \end{cases}$$

$$x_{ij} \geq 0, \quad i = \overline{1, m}, \quad j = \overline{1, n}$$

(1.4)

If the available total $(D = \sum_{i}^{m}$is equal to the necessary total $(N = \sum_{j}^{n}$, then the transportation problem is balanced; if not, when D<N or D>N, the problem is unbalanced.

3. THE CONCEPT OF DELAY IN THE FORRESTER TECHNIQUES

In the Forrester approaches, the description of some systems through equations highlights the way of use of any state of the system in calculating the future state in which the system will be after a period of time. The equations describe how the state of the system changes over time and how these changes are gathered step by step to reveal the behavior of the system; therefore the equations, i.e. the instructions to calculate the values in the next step are named simulation model, which is used instead of the real system.

Inside the system there are interactions which result in their growth, fluctuation and change. The system's behavior is produced by a combination of its interacting components that operate inside a limit that defines the system. In figure 1 we are shown the design of a system with reverse negative reaction, which autocorrects itself by reducing and contracting the deviation between the real and the necessary state.

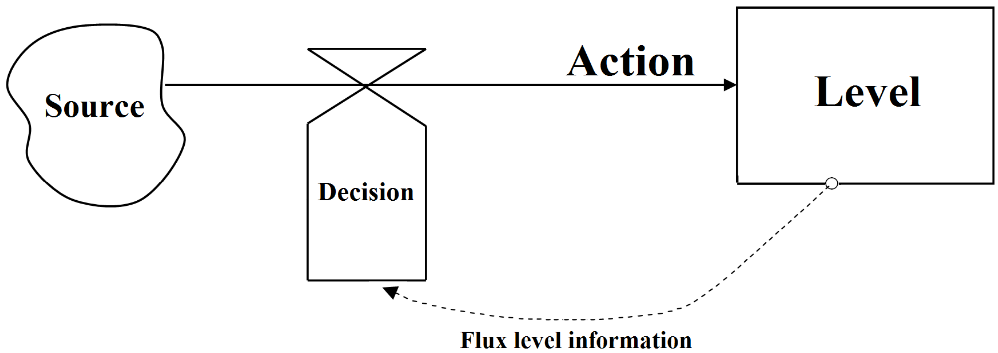

Figure 1. Reverse negative reaction system

Source: Forrester, J., *System principles. Theory and self-training programmed*, Tehnical Publishing House, București, 1979

We can observe that the reverse connection loop couples the decision, action, system level and information. The decision is based on the available information, commanding an action that influences the system level. A decision process can be part of more than one reaction loops.

Complex systems are ensembles of reaction loops in interaction. The system hierarchies can be resumed as so:

- levels and rhythms (strategies) as substructures of the reverse connection loop;
- objective, state, difference, action, as substructures of rhythms.

The following statements regarding the Forrester techniques are necessary:

- the levels cumulate the difference between the rhythms of the entry flux and the exit flux. The levels describe the state of the system at a given time, being represented by the level equations; the rhythms show how quick the levels change, being represented by the rhythm equations;
- the level and rhythm variables need to alternate, any path in the structure of a system containing alternatively variable of rhythm and variables of level;
- the informational links connect the levels to the rhythms;
- the reverse connection loop of the first order has an allure that is exponential in time, and the time constant of the loop links the levels to the rhythms;
- the analysis of the system's dynamic behavior is done on the following six fluxes: commands, materials, money, personnel, machinery and information.

In the dynamic models, delays constitute highly important elements and useful to shaping the transformation processes which entail a certain duration and diverse transformations of the entries into exits. Basically there are delays on all the known flux paths, in the form of material delays, financial, machinery, personnel, commands, and informational delays, suitable to the informational flux.

A delay is actually a process of conversion, that accepts a rhythm of given entry flux and supplies a rhythm of flux resulted as an out. The time delay from the flux of physical or informational quantities can be created by combining the level and the rhythm equations. An exponential delay of the first order is the structure formed from a rhythm and a level in which the exit rhythm is dependent only on the previous level and the time constant that expresses the duration of the delay. So, the exponential delay of the first order will be made up by a simple level (that absorbs the difference between the entry and exit rhythms) and a rhythm of the exit rhythm, which depends on the level of average delay (a constant) as shown in figure 2.

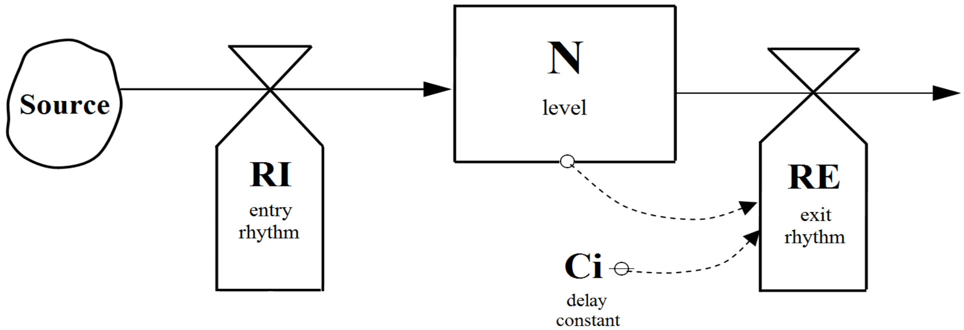

Figure 2 The exponential delay model

Source: Forrester, Jay, *System principles. Theory and self-training programmed*, Tehnical Publishing House, București, 1979

The exit rhythm RE will be equal with the level divided by the delay constant (average delay):

$$R = N/CI \qquad (1.5)$$

The representation of the delay is not complete if there is no equation to generate the internal level of the transit unit, N, The N level, included in the delay, is an accumulation of the difference between entry and exit:

$$N_1 = N_0 + (RE - RE) \qquad (1.6)$$

The level equation and the rhythm equation transforms the entry rhythm RI into a delayed rhythm RE.

In order to get material delays of the greater order, a number of "n" cells of delay of the first order is coupled. This number of coupled cells is the order of the resulted delay. Usually, in dynamic models, a delay of the third order or of a multiple of three is used.

4. THE SHAPING OF THE ROAD TRAFFIC AS A REPLY TO THE DELAYS IN THE FORRESTER DYNAMIC MODELS

The conception of the delay in the Forrester techniques is the starting point in the problem of shaping the road traffic.

Heavy traffic on certain portions of the road and normal traffic on others, generates delays that are that are compensated in the production process. For the realization of this process in normal conditions, each material resource needs to be present at the time and place in which it takes place in the required quantity and at the quality and dimensions established through technological prescriptions. These aspects are required also in the interactions between the market's demands and the different products made by the company and the means of distributing them.

Such a model of road traffic entails from the get go certain simplifications, in the sense that it will entail that road traffic is realized on a single lane, and that on the monitored road are no intersections, refuges, or other entries or exits so that the number of vehicles remains constant over time. We consider that easy/heavy traffic entails high/low speed and the small/large number of cars on a certain portion of the road is translated in low/high density in the number of cars. In establishing the variables of the model that describe the evolution of the analyzed system, entails that these are continuous and differentiable space-time functions. The independent variable will be time (t) and the special coordinate (x). The dependent variables are the velocity at which traffic occurs:

$$v(x,t) = \frac{dx}{dt}, \qquad (1.7)$$

And the traffic density, i.e. the number dN of cars per measuring unit dx, measured over the road:

$$\rho(x,t) = \frac{dN}{dx} \qquad (1.8)$$

Because the number of cars is preserved, the equation of the evolution of the number of cars can be written as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0 \qquad (1.9)$$

Because the problem has two dependent variables, ρ and v, in order to solve it, one more equation is needed to link the two variables. By analyzing the problem, the link between the traffic velocity and its density is translated by the fact that where the traffic is light, the speed will be higher and vice-versa. Under these conditions, the simplest relation between the v and ρ is a linear type:

$$v = \alpha - \beta \rho \qquad (1.10)$$

α and β are determined using the following conditions:

1. when the density of the traffic is very low ($\rho \to 0$) the speed is the legal max for that type of road v_{max}. Under these conditions it results that $\alpha = v_{max}$.
2. when traffic density is high, $\rho = \rho_{max}$, the speed is minimal ($v = v_{min}$). so it results that:

$$\beta = \frac{v_{max} - v_{min}}{\rho_{max}} \qquad (1.11)$$

In the end, the model is complete if an initial condition is known, for example:

$$\rho(t = 0) = \rho_0 \qquad (1.12)$$

By replacing relation (1.10) in (1.9) it results:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\alpha \rho - \beta \rho^2) = 0 \qquad (1.13)$$

The most reasonable assumption is that the traffic variations take place as a result of the action of some random disturbances in the homogeneous traffic.

We consider the homogenous state of traffic given by:

$$\rho = \rho_0$$
$$v_0 = \alpha - \beta \rho_0 \qquad (1.14)$$

By slightly disturbing this state $\rho = \rho_0 + \rho_1,\ \rho_1$ then the equation (1.13) can be linearized and the equation which generates the space-time evolution of the disturbance is:

$$\frac{\partial \rho_1}{\partial t} + (\alpha - 2\beta\rho_0)\frac{\partial \rho_1}{\partial x} = 0 \qquad (1.15)$$

By admitting that the disturbance is of the form of a plane monochromatic harmonic wave, the speed at which this disturbance wave propagates is:

$$c = \frac{\omega}{k} = \alpha - 2\beta\rho_0 \qquad (1.16)$$

where ω is the pulsation and k is the wave number.

Comparing (1.14) with (1.16) it can be seen that $c < v_0$. This means that, if a disturbance in traffic is produced, then all the cars will reach the front of the disturbance which propagates, thus explaining the formation of traffic jams and by default the generation of delays modeled by Forrester. More so, at the encounter of the disturbance, the cars will perceive it as a shock, which will determine the drivers to break. If $\rho_0 > \frac{\alpha}{2\beta}$, then from the equation (1.16) it can be seen that $c < 0$ and also $v_0 < \frac{\alpha}{2}$, which means that the disturbance propagates backwards and the cars are moving with a speed lower than half the legal limit. This happens when one of the drivers suddenly breaks or has an accident. In essence, any delay in traffic generated by the causes specified above can be modeled through the Forrester techniques, in the purpose of understanding it.

4. CONCLUSIONS

Modeling transport processes refers both to shaping and stirring but also to maintaining the material fluxes imposed by the relations of the company with the outside environment, in correlation with the informational fluxes specific to them. Into account are taken the relations that take place upstream of the company, with the suppliers of the needed material resources and also the ones that take place downstream, with the distribution networks and the opening markets. Starting from the modeling of delays through the dynamic Forester models, it was attempted to elaborate a model of road traffic that will contain the relevant features of the studied phenomena. This was accomplished through the simplification and essentialization of all relevant aspects, resulted through observing the phenomena, with establishing the purpose, resources and desired resolution. The starting point was identifying the variables responsible in the evolution of the studied phenomena, sorting these into variables that are considered relevant or irrelevant, followed by establishing the mathematical relation between them. Using this model, we can formulate explanations for the studied phenomena and predictions for the evolution of traffic in the company's logistic.

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