A GENERAL MODEL TO STUDY THE PROBLEM OF PHASING, IN TIME AND SPACE, THE EXECUTION OF TECHNOLOGICAL OPERATIONS USING THE THEORY OF GROUPS

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Abstract: *This study on production phasing starts from the basics of the theory of groups. The authors highlight the correlations between symmetries and permutations, as a starting point in determining the production phasing using the group of permutations. We developed a general model to study the phased execution of the technological operations using simple groups, as basic building blocks of Galois theory, and examining the symmetry properties of permutations and group isomorphism.*

Key words*:* production phasing, group, subgroup, group isomorphism, permutations, symmetries, Galois theory.

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1. THE GENERAL PROBLEM OF PHASING LAUNCHES IN PRODUCTION

Phasing is a problem of production programming that refers to arraging the operations gradually, in time and space. The concept of production phasing means to distribute ***n*** production tasks on ***m*** machines (performers), based on some criteria rigorously established. Whatever type of production, phasing experinces difficulties both in achieving the objective (filling positions entirely and minimizing the production cycle) and in directing resources needed for production.

The possible formulations for the optimization criterion (the performance index) are:

- minimizing the production cycle (the total duration of operations);
- minimizing the stagnation time of machines;
- minimizing uncomplete production;
- minimizing specific costs;
- minimizing delays to delivery deadlines of products.

The problem of phasing may be stated statically (if all data are known in advance) or dynamically (in case of data reffering to products, resources and disturbances may change during the production). The problems of dynamic phasing require a methodology for solutions in real time.

When formulating the problem of phasing, it is essential to select the correct optimization criterion (the performance index), because the importance of various criteria – from the econonomic and production points of view – is not the same. The applications made so far show that the efficiency achieved by shortening the average cycle of a model production is much smaller than the efficiency achieved by reducing the stagnation of machines. Even minimizing the production cycles or the stagnation of equipments are considered to be measures of secondary importance compared to the rhythmic supply with parts of fitting departments. A

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comparative study conducted through simulations, in terms of efficiency of some optimal criteria in production phasing, led to the conclusion that an optimal economic solution is the uniform charging of the equipment (not maximum), without gaps and jams (Moldoveanu,G., Dobrin,C., 2003).
The general problem of phasing states as follows (Constantinescu, D., 2003):
Given:

a) a number of product lots, each consisting of a number of parts;
b) the technological process for each lot consisting of a number of operations, their sequence being known;
c) the durations of operations, the preparation time and the transport time between two operations;
d) the groups of available machines, the number of machines in each group and the terms of release.

Find: that sequence of lot processing on each machine that provides the best loading of machines (the times of inactivity between two lots should be minimal).

2. ELEMENTS OF THE THEORY OF GROUPS NEEDED IN APPROACHING THE PROBLEM OF PRODUCTION PHASING

In mathematics, a group is an algebraic structure consisting of a set defined by a law of internal composition (operation) that combines two elements of that set to form the third element of that set. To be a group, the range and the operation should meet a number of conditions. For example, the group G together with a binary operation denoted $\cdot$ G x G $\rightarrow$ G, (x, y) $\rightarrow$ x $\cdot$ y meets the following axioms (Buşneag, D., 1994):

(G1) (Associativity) $(x \cdot y) \cdot z = x \cdot (y \cdot z)$, $(\forall)$ x,y,z G;

(G2) (Neutral element) $(\exists)$ $e \in G$ so that $x \cdot e = e \cdot x = x$, $(\forall)$ $x \in G$;

(G3) (Reversibility) $(\forall)$ $x \in G$, $(\exists)$ $x^{-1} \in G$ so that $x \cdot x^{-1} = x^{-1} \cdot x = e$

If in addition it occurs:

(G4) (Commutativity) $x \cdot y = y \cdot x$, $(\forall)$ $x,y \in G$, say that G is abelian or commutative.

Although these are properties common to many mathematical structures, such as sets of numbers, formulating axioms is detached from the concrete nature of the group and the respective operation. This allows us to handle some entities of different mathematical origins in a flexible manner, keeping the essential structural issues common to many types of objects. The omnipresence of groups in numerous areas, both mathematical and others, makes them a central organizing principle in the contemporary mathematics.

Groups have the fundamental property of symmetry. A group of symmetry abstracts the symmetry characteristics of a geometric object: it consists of a set of changes that does not modify the object and the operation of combining these changes by chaining them. Such symmetry groups, especially the continuous Lie groups, play an important role in many academic subjects.
For this study, we recall some basic mathematical knowledge on permutations and groups (Barbilian, D.,1988):
a) the analyzed permutations will be characterized by:

- the transposition of the permutation τ, denoted by $\tau = (i,j)$, for which *i,j* form the set $A = \{1,2,\ldots,n\}$, $i \neq j$, resulting $\tau(i) = j$, $\tau(j) = i$ şi $\tau(k) = k$, $(\forall)$ $k \in A \setminus \{i,j\}$;
- the inversion of a permutation σ, denoted by $Inv(\sigma)$, if the ordered pair (i,j), with $1 \leq i < j \leq n$ we have $\sigma(i) > \sigma(j)$. The sign (signature) of permutation will be $\varepsilon(\sigma) = (-1)^{Inv(\sigma)}$, for which the value +1 represents the even permutation and the value represents -1 the odd permutation;

- the order of a permutation α, denoted by ord α, which states that if $\alpha \in S_n$ is an m- cycle, then ord α = m.

b) in studying groups, we considered the following elements of analysis:

- the neutral element of a subgroup of a group coincides with the neutral element of the group it belongs to. For any element of the subgroup, its inverse in this subgroup coincides with its inverse in the group;
- the order of any subgroup of a finite group is a divisor of the group order (Lagrange's theorem). It results that within a finite group, the order of each element is finite and it is a divisor of the group order, and every group of even order is cyclic (the group is generated by a one of its elements);
- the composition factor is the ratio between the order of the parent group and the order of the subgroup;
- if (G ·) is a group and H is one of its subgroups, then the following statements are equivalent:
  a) $xH = Hx$, $(\forall)\ x \in G$
  b) $(\forall)\ x \in G$, $(\forall)\ h \in H \Rightarrow xh^{-1} \in H$

The subgroup H of the group G that satisfies the equivalent statements is called a normal subgroup (normal divisor) of the group G.

- any finite group is isomorphic to a subgroup of a group of permutations (Cayley's theorem);
- for two groups $(G,\circ)$ and $(G',\bullet)$ a function $f : G \to G'$, with the property $f(x \circ y) = f(x) \bullet f(y)$, $(\forall)\ x,y \in G$ is called a group morphism. A function $f : G \to G'$, which is a morphism of groups is reversible and the inverse function $f^{-1} : G' \to G$ is also a morphism of groups, called group isomorphism. If between the two groups there is at least one isomorphism, we say that the groups are isomorphic ($G \approx G'$).

It is important for this study that the direct and inverse isomorphisms transport a structure in the other, thus transferring all the properties of a group in the other group, as we exemplify in section 3. Therefore, two isomorphic groups have the same properties, so they are basically "identical" in terms of algebraic behavior. Therefore in algebra, the "recognition" of a group is made by highlighting a known group isomorphic with it. Thus we reach the following step of abstraction: all groups isomorphic between them behave identically, they can be assimilated with one of them, detaching from the set and operation of each group taken individually, no matter their specific identity.

3. THE CORRELATION BETWEEN SYMMETRIES AND PERMUTATIONS THROUGH THE THEORY OF GROUPS

A symmetry of a mathematical object is a transformation that preserves the structure of the object; we note first that symmetry is a process, not an object, and a permutation is a way to rearrange things. It is not, strictly speaking, rearranging itself, but the rule to be applied to obtain rearrangement.

There are three key words in defining a symmetry: "transformation", "structure" and "invariance". The *transformation* refers to the various things we can do with a particular object, while the *structure* consists of the signifiant mathematical properties; the *invariance* means that the structure of the transformed object is the same with that of the original. Permutations and groups are intimately linked; in fact, the group concept was born from the study of permutations.

Let us examine the set of all possible permutations of numbers 1,2,3, knowing that the number of permutations of n different objects is achieved with n! :

$$\begin{pmatrix}123\\123\end{pmatrix} \quad \begin{pmatrix}123\\231\end{pmatrix} \quad \begin{pmatrix}123\\312\end{pmatrix} \quad \begin{pmatrix}123\\132\end{pmatrix} \quad \begin{pmatrix}123\\321\end{pmatrix} \quad \begin{pmatrix}123\\213\end{pmatrix}$$

$$\mathbf{I} \quad \mathbf{s_1} \quad \mathbf{s_2} \quad \mathbf{t_1} \quad \mathbf{t_2} \quad \mathbf{t_3}$$

The element giving the identical numerical order is denoted by I. Each of the operations t1, t2 and t3 implements and interchanges two of the numbers, leaving the third in its place. The two operations s_1 and s_2 are both cyclical permutations, moving the numbers around a circle. Let's see what happens if we apply two successive permutation operations, taking into account that it is important which number replaces other, and not the order in which they are written. Take, for example, t_1 followed by s_1. The operation t_1 leaves 1 in its place, then t_1 changes 1 in 2. The net result is the transformation $1 \rightarrow 2$. At the same time, t_1 replaces 2 with 3, then s_1 replaces 3 with 1, causing the net result $2 \rightarrow 1$. Finally, 3 is transformed into 2 by operation t_1 and then back into 3 by operation s_1. We find that t_1 followed by s_1 gives the permutation:

$$\begin{pmatrix}123\\213\end{pmatrix}$$

which is precisely the operation t_3. In other words, if the symbol $^{\circ}$ designates the operation "*followed by*", we find that $s_1 \circ t_1 = t_3$, considering that the first applied operation is always the right one. Judging similar for the other operations, the graph of the composition law (Cayley's graph) has the following form:

$\circ$	I	s_1	s_2	t_1	t_2	t_3
I	I	s_1	s_2	t_1	t_2	t_3
s_1	s_1	s_2	I	t_3	t_1	t_2
s_2	s_2	I	s_1	t_2	t_3	t_1
t_1	t_1	t_2	t_3	I	s_1	s_2
t_2	t_2	t_3	t_1	s_2	I	s_1
t_3	t_3	t_1	t_2	s_1	s_2	I

Note that the set of all permutations of three elements forms a group. In fact, this statement is true for the permutations of any number of elements. The graph demonstrates both the closing (the composition of any two permutations of three elements gives another permutation of three elements), and the fact that each permutation corresponds to a symmetrical one (which cancels the effect of the first). One may notice easily that s_1 and s_2 are symmetrical (applying the two transformations one after another, the original order restores: $s_1 \circ s_2 = I$; $s_2 \circ s_1 = I$). Similarly, each of the operations t_1, t_2, t_3 is its own symmetrical item; applying it on any of them twice, the status quo recovers: $t_1 \circ t_1 = I$; $t_2 \circ t_2 = I$; $t_3 \circ t_3 = I$. The order of the permutation group, which is the natural number equal to the number of items in the group is 6 and is denoted by $|G|$.

We intend to further analyze the symmetries of an equilateral triangle and show that the group of permutations of three elements and the symmetry group of the equilateral triangle are isomorphic. We find that there are six such structures that leave the triangle unchanged: they correspond to the neutral element, the rotation with 120^0, the rotation with 240^0 rotation, and reflection in relation to the three axes. What is actually doing a 120^0 rotation of the triangle, trigonometrically? It only takes the edge A and moves it from position 1 to position 2. At the same time, it moves the edge B from position 2, and the edge C from position 3 to position 1. In other words, we see this rotation as a permutation of the positions 1,2,3 in relation to the edges of the triangle rotating:

$$\begin{pmatrix} 123 \\ 231 \end{pmatrix}$$

Analogically, each of the other five symmetries of the triangle corresponds to one of the other permutations, meaning that the structure of the two groups is identical. Therefore, the two groups, the group of permutations of three elements and the symmetry group of the equilateral triangle, are isomorphic. If we consider the three elements of the group of permutations to be three technological operations, then this group is also isomorphic to the group of symmetries of an equilateral triangle.

Besides the isomorphism mentioned above, let us present another that highlights how two other groups have an identical structure. Let us consider four classes of technological operations denoted A, B, C, D, each with two subclasses bearing the indices 1 and 2 as follows:

$$\begin{tikzcd}[row sep=small] & A_1 & & B_1 & & C_1 & & D_1 \\ A \arrow[ur] \arrow[dr] & & B \arrow[ur] \arrow[dr] & & C \arrow[ur] \arrow[dr] & & D \arrow[ur] \arrow[dr] & \\ & A_2 & & B_2 & & C_2 & & D_2 \end{tikzcd}$$

We require the following restrictions:

1. Class A of technological operations can be performed simultaneously with the operation class C
2. Class B of operations can be performed simultaneously with the operation class D
3. The technological operations resulted from the subclasses A_1 and C_2 represent class D
4. The technological operations resulted from the subclasses C_1 and A_2 represent class B
5. The technological operations resulted from the subclasses B_1 and D_2 represent class C
6. The technological operations resulted from the subclasses D_1 and B_2 represent class A

The relations (1) and (2) may be represented by the following correspondence, denoted by f:

$$f = \begin{pmatrix} ABCD \\ CDAB \end{pmatrix}$$

We note that if this permutation is performed twice, it restores the original idea $f \circ f = I$ (where I is the identity), f brings A instead of C and C instead of A, so that applying f twice returns A under itself, doing the same for the other letters. Rules (3) - (6) may be represented by two permutations, denoted by p and m:

$$p = \begin{pmatrix} ABCD \\ DCBA \end{pmatrix} \qquad m = \begin{pmatrix} ABCD \\ BADC \end{pmatrix}$$

We note again that $\circ\, p \circ p = I$ and $m \circ m = I$. Also, each of the permutations f, p, m, operating in succession, produces the third one. Cayley's graph looks as follows:

∘	I	f	p	m
I	I	f	p	m
f	f	I	m	p
P	p	m	I	f
m	m	p	f	I

Let us analyze another group and study the isomorphism with the above. Let us consider other four simple sets: set X has only one element, which is U1 machine; set Y has one element, the machine U2; set Z is composed of two elements: the machines U1 and U2. I designates the empty set, which has absolutely no element, its role is similar to that played by zero in the ordinary addition. Let us use the symmetric difference of sets to combine any two of these sets, according to the relation:

$$A \Delta B = (A \setminus B) \cup (B \setminus A)$$

Cayley's graph is the following:

Δ	I	X	Y	Z
I	I	X	Y	Z
X	X	I	Z	Y
Y	Y	Z	I	X
Z	Z	Y	X	I

One may note that each of X, Y, Z is its own symmetrical item. Therefore, associating the operation Δ to the sets X, Y, Z, I, we get a group. Although both groups' members in the two cases and laws of composition are completely different, the two groups have an identical structure, being isomorphic.

4. SETTING THE SEQUENCE OF TECHNOLOGICAL OPERATIONS THROUGH THE GROUP OF PERMUTATIONS. THE IMPORTANCE OF GALOIS THEORY IN SOLVING THE PRODUCTION PHASING

We approach the processing sequence through the group of permutations in two ways, identifying a distinct goal for each mode, as follows:

- in case of a first approach in terms of groups of different cost technological operations, we intend to determine the success probability of sequencing the operations based on a strategy that uses the information from a finite number of steps;

- in a second approach, we intend to establish an execution sequence of technological operations based on normal subgroups within the group of permutations S_3. Galois theory, given the phasing expressed by an equation, could highlight a formula to solve it through Galois group.

According to the the first approach, let us consider four groups of technological operations whose costs are denoted as follows:

1- high cost operation

2- average cost operation

3- low cost operation

4- lowest cost operation

The operations are not performed simultaneously and there are no returns to a previous, inadequate operation. For the four groups of operations, each of the 4! = 24 permutations of the execution sequence has the same probability, as in the table below:

1234	2134	3124	4123
1243	2143	3142	4132
1324	2314	3214	4213
1342	2341	3241	4231
1423	2413	3412	4312
1432	2431	3421	4321

For example, the sequence 3142 means that the low-cost operation is first executed, followed by the high cost operation, then the lowest cost operation, and the last operation is the average cost one. The question is what are the maximum chances of programming the low cost operation. The strategy to approach this problem, based on the information already gathered, is this: we choose a number to call k, between 1 and 4; after we have researched k-1 operations, we opt for the first one which is better than all others. Using this strategy, not telling what value to choose for k, we have to find the value of k that gives the highest probability to choose the lowest cost operation, as follows:

- for k = 1 (k-1 = 0), the first operation is the one that ends up being chosen, based on the six permutations of the sequnce in which 4 occurs first: 4321, 4312, 4231, 4213, 4132, 4123. The probability to run one of the six permutations of the twenty-four possibilities is one of four existing;
- for k = 4 (k-1 = 3), it is considered the chance that the fourth and last operations to be better than any of the previous three. The situation corresponds to the permutations 3214, 3124, 2314, 2134, 1324, 1234, the probability is that the order of operation execution is still one in four;
- for k = 3 (k-1 = 2), we choose two operations, then the first that comes after these two and it is better than both. The permutations in this case leading to the lowest cost operation are: 3241, 3214, 3142, 3124, 2341, 2314, 2143, 1342, 1324, 1243. In this case, there are ten permutations that lead to the best choice, while the chances of success are 10:24 or about 42%;
- for k = 2 (k-1 = 1), the option will be the first operation which has the cost lower than first appeared. The permutations are: 3421, 3412, 3241, 3214, 3142, 3124, 2431, 2413, 2143, 1432, 1423. Note that this is the best strategy to be adopted since the desired result in 11 of the 24 possibilities offers a success probability of about 46%. We demonstrate mathematically that if the number of operations is higher than 30, the "rule of 37%" produces the best chances of success, meaning that we examine 37% of the possibilities and then we choose the lowest cost operation of all operations taken before.

According to the the second approach of using an equation in production phasing, we examine first the symmetry properties of the permutations in the equation solutions. We demonstrate using Galois Theory that the equation has its own profile symmetry, representing its symmetry properties. Galois group would represent the largest group of permutations of the solutions, assumed to exist, and leaving unchanged the values of certain combinations of these solutions. For any degree there are equations that have the maximum possible symmetry, meaning that Galois group is the whole S_n. Galois defined some particular subgroups, called normal, that have been defined in section 2. If a group has no normal subgroups (other than the two trivial subgroups, one made only of the identical transformation and the other represented by the group itself), then it is called simple. All groups may consist of simple groups, but the simple groups can not be decomposed further through a similar process (Sâmboan, G.,1968).

Based on Cayley's graph described in section 3, the group of six permutations of three elements S_3 has the following subgroups:

- [I, s_1,s_2], [I, s_2, t_3], denoted by T and U, respectively, containing three permutations each;
- [I, t_2], [I,s_1], [I,t_1], denoted by V,W,X, containing two permutations each;
- [I], denoted by I ;I containing one permutation.

According to the definition of the normal subgroup in section 2, it results that the normal subgroups are T, U, I, as well as the whole group [I, s_1, s_2, t_1, t_2, t_3]. The subgroups V, W, X, containing two permutations each, are not normal. To demonstrate this, for example for the subgroup T, we take a member of it and multiply it to the left with a member of S_3, and to the right with the symmetrical element of the element selected from S_3, resulting: $t_1 \circ s_1 \circ t_1 = t_3 \circ t_1 = s_2$, which is an element of T. Things are similar when choosing t_2 in S_3, where its symmetrical is again t_2 (see Cayley's graph), as follows: $t_2 \circ s_1 \circ t_2 = t_1 \circ t_2 = s_2$.

The permutations of T normal subgroup are characterized by the following values of inversion, signature, transposition and order, as follows:

$$\begin{pmatrix}123\\123\end{pmatrix} \qquad \begin{pmatrix}123\\231\end{pmatrix} \qquad \begin{pmatrix}123\\312\end{pmatrix}$$

I	**s_1**	**s_2**
$Inv(I) = 0$	$Inv(s_1) = 1+1=2$	$Inv(s_2) = 2+0 =2$
$\varepsilon\,(I) = (-1)^0 = 1$(even)	$\varepsilon\,(s_1) = (-1)^2 = 1$(even)	$\varepsilon\,(s_2) = (-1)^2 = 1$(even)
$\tau_I = 0$	$\tau_{s1} = 0$	$\tau_{s2} = 0$
ord I = 0	ord $s_1 = 3$	ord $s_2 = 3$

It results three variants of processing in the sequence of operations, given by:

3 - 1 - 2
2 - 3 - 1
1 - 2 - 3

for which the option for a technological variant is given by the cost of each operation.

Similarly, for U normal subgroup it results:

$$\begin{pmatrix}123\\123\end{pmatrix} \qquad \begin{pmatrix}123\\312\end{pmatrix} \qquad \begin{pmatrix}123\\213\end{pmatrix}$$

I	**s_2**	**t_3**
$Inv(I)=0$	$Inv(s_2) = 2+0=2$	$Inv(t_3) = 1+0 = 1$
$\varepsilon\,(I) = (-1)^0 = 1$(even)	$\varepsilon\,(s_2) = (-1)^2 = 1$(even)	$\varepsilon\,(t_3) = (-1)^1 = -1$(odd)
$\tau_I = 0$	$\tau_{s2} = 0$	$\tau_{t3} = (1,2)$
ord I = 0	ord $s_2 = 3$	ord $t_3 = 2$

The variants of processing, in the sequence of operations, are:

2 - 1 - 3
3 - 1 - 2
1 - 2 - 3

This reasoning may be pushed forward, namely the possibility of solving the problem of production phasing through a determined formula. Galois Theory gives again an answer to this problem: the equations formulating the production phasing should have a solvable particular group (Galois group), meaning that each of the composition factors generated by its normal sub-maximal subgroups is a prime number. Let us calculate the composition factors (f.c) (Năstăsescu, C., Niţă, C., Vraciu, C.,1986):

S_3 and T (respectively S_3 and U) => f.c = 6:3 = 2
T and V (respectively T and W; T şi X) => f.c = 3:2 = 1,5
T şi I => f.c = 3:1 = 3

It results that the hierarchy of generations of subgroups S_3 and T, T and I gives the row of composition factors 2, 3, which are prime numbers. If the problem of phasing is expressed by an equation of third degree, it would have a maximum symmetry if Galois group is S_3, which is solvable because both composition factors are prime numbers. Things are similar for the fourth degree of the equation, but for a five degree equation a formula is not possible. It is known that an equation can be solved by a formula provided that its Galois group is solvable. The explanation lies in the fact that, although there might be equations for which Galois group is S_5, it is not solvable because one of its composition factors is 60, which is not a prime number.

4. CONCLUSIONS

The problem of production phasing represents most part of the operational research, giving, through algorithms, solutions for optimal sequence of technological operations. In this paper, we tried to move the center of gravity of the problem in the theory of finite groups of permutations, that may provide well determined and mathematically justified solutions. The concept of symmetry is important in this study, since we approach the correlation between symmetries and permutations through the theory of groups, starting from their omnipresence in all areas, both within and beyond mathematics. It has been demonstrated that the finite groups are immersed in the groups of permutations, meaning that the study of finite groups reduces to that of subgroups of permutation groups. The isomorphism of the technological operation group with the group of permutations of three elements S_3 reveals that, on the one hand, that the group operations is cast in the same "mold" as the group of permutations, and on the other hand, their algebraic behavior is identical in case of their commutativity and cyclicity. We started from the idea that the specific bijective of isomorphism "transfers" of a group structure to an "amorphous" (unstructured) set and, conversely, any structure can be thought of as coming from such a "transport" conducted by a bijection.

The sequence of technological operations is established through the analysis of normal subgroups, calculating transpositions, inversions and permutation order specific to these subgroups. Galois theory may find applicability in solving the problem of production phasing, but since there is no specific equation it can not be established yet a determined formula, although Galois group is solvable in some cases by the row of composition factors prime numbers. The problem is still under study, and we look for algebraic methods of solving the problem in the future.

REFERENCES

1. Barbilian, D., *Algebră [Algebra]*, Bucharest: Didactică şi Pedagogică, 1985.
2. Barbilian, D., *Algebră axiomatică [Axiomatic Algebra]*, Bucharest: Didactică şi Pedagogică, 1988.
3. Bourbaki, N., *Algebre [Algebra]*, chap.1-9, Paris: Herman.
4. Buşneag, D., *Teoria grupurilor [The Theory of Groups]*, Craiova: Universitaria, 1994.
5. Clifford, A., Preston G., *The Algebraic Theory of Semigroups*, vol.I,II, Mathematica Surveys, 1967.
6. Constantinescu, D., *Managementul operaţional al producţiei [The Operational Management of Production]*, Craiova: Sitech, 2003.
7. Constantinescu, D., Moldoveanu, G., *Conducerea operativă a producţiei [The Operational Management of Production]*, Editura Craiova: Ecmondo, 1994.
8. Jaba, O., *Gestiunea producţiei şi operaţiilor. Metode şi tehnici ale managementului operaţional al producţiei [Production and Operation Management. Methods and Techniques of Production Operational Management]*, Bucharest: Economică, 2002.
9. Kurosh, A.G., *Higher Algebra*, Moscow: Mir Publishers, 1972.

10. Kurosh, A.G., *Teoria grupurilor [The Theory of Groups]*, Bucharest: Tehnică, 1959.
11. Lang,S., *Algebra*, Addison – Wisley Publishing Company, 1965.
12. MacLane, S., Birkhoff, G., *Algebra [Algebra]*, New York: Macmillan Company, 1967.
13. Moldoveanu,G., *Managementul operaţional al producţiei [The Operational Management of Production]*, Bucharest: Economică, 1999.
14. Moldoveanu,G., Dobrin,C., *Managementul operaţional al producţiei. Studii de caz [The Operational Management of Production. Case Studies]*, Bucharest: Economică, 2003.
15. Moldoveanu, G., *Metode actuale de management operaţional [Current Methods of Operational Management]*, Bucharest: MatrixRom, 2003.
16. Myller Lebedev, V., *Lecţii de algebră [Algebra Lessons]*, Bucharest: Academiei, 1953.
17. Năstăsescu, C., Niţă, C., Vraciu, C., *Bazele algebrei [Basics of Algebra]*, Bucharest: Academiei, 1986.
18. Nemeti, L., *Programarea în timp a fabricaţiei [Production Timing]*, Cluj- Napoca: Dacia, 1975.
19. Nemeti, L., *Metode euristice de ordonanţare a fabricaţiei [Heuristic Methods of Production Phasing]*, Cluj- Napoca: Dacia, 1977.
20. Postnikov, M., *Teorie Galois [Galois Theory]*, Moscow, 1963.
21. Purdea, I., Pic, Gh., *Tratat de algebră modernă [Treaty of Modern Algebra]*, vol I,II, Bucharest: Academiei, 1982.
22. Sâmboan, G., *Teoria lui Galois [Galois Theory]*, Bucharest: Tehnică, 1968.
23. Ştefănescu,M., *Teoria grupurilor [The Theory of Groups]*, Iaşi: Universităţii "Alexandru Ioan Cuza", 1993.
24. Tofan, Volf, A.C., *Algebră. Inele. Module. Teorie Galois [Algebra. Rings. Modules. Galois Theory]*, Bucharest: MatrixRom, 2001.
25. Van der Waerden, *Moderne Algebra*, Berlin: Springer Verlag, 1937.